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RESPONSE OF MULTIPLY SUPPORTED SECONDARY SYSTEMS TO EARTHQUAKES IN FREQUENCY DOMAIN

APARNA DEY*† AND VINAY K. GUPTA‡
Department of Civil Engineering, Indian Institute of Technology, Kanpur 208016, India

SUMMARY

A new formulation of the transfer function has been proposed for the seismic analysis of linear, multiply supported secondary systems. The transfer function for a given response quantity has been formulated by directly using the fixed-base modes of the primary and secondary systems. This approach is exact and does not involve the determination of the combined system properties. Further, it is applicable to the secondary systems with various mass ratios and configurations. A few example primary–secondary systems have been considered to illustrate the proposed formulation in case of different mass ratios. It has also been shown how the proposed formulation can be used to obtain reasonably accurate stochastic estimates of the secondary system responses. © 1998 John Wiley & Sons, Ltd.

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KEY WORDS: multiply supported secondary systems; seismic analysis; stochastic response; PSDF-based approach; transfer function; fixed-base modes

1. INTRODUCTION

In several civil engineering structures, e.g. power plant installations, industrial structures, etc., it is critically important to ensure the seismic safety of secondary systems like piping and equipment for their continued performance immediately after the earthquakes. It is generally considered inappropriate to generate floor response spectra for the seismic qualification of these secondary systems when those are attached to different degrees of freedom of the primary system. These systems can be analysed directly by considering the combined P–S system as a single dynamic unit, by determining the eigenproperties of the combined system via state-space approach of Foss,[1] and then by employing a modal analysis (e.g. see References 2–7). However, since the mass and stiffness properties of the primary and secondary systems are widely different, there may be ill-conditioning of matrices leading to numerical difficulties. Moreover, it may sometimes be necessary to consider a number of design alternatives of the secondary system and then, such an analysis would involve the evaluation of the dynamic properties of the combined system several times.

To avoid the problems involved in the direct methods, the real eigenproperties of the classically damped sub-systems have also been used in the evaluation of the properties of the combined system. For example, Gupta and Jaw[8] have presented a method in which a complex mode is replaced by two real modes. In the mode synthesis approach proposed by Suarez and Singh,[9] the primary and secondary systems are sequentially coupled by considering one support at a time. In this procedure, the eigenproperties of the partially connected system have to be determined at each coupling stage. Another formulation based on the component-mode synthesis approach has been presented by Muscolino,[10] wherein the reduced equations of motion have to

* Correspondence to: V. K. Gupta, Department of Civil Engineering, Indian Institute of Technology, Kanpur-208016, India. E-mail: vinaykg@iitk.ernet.in
† Formerly Graduate Student
‡ Associate Professor

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be solved by using the state-space approach. Perturbation techniques (e.g. those by Sackman and Kelly,[11] Sackman *et al*.,[12] Hernried and Sackman,[13] Igusa and Der Kiureghian,[14] Singh and Suarez,[15] and Perotti[16]) have also been used to calculate the combined system properties. Though these methods have been formulated for those cases where the secondary system is considerably lighter than the primary system, those have yielded physically meaningful parameters with closed-form expressions. For example, Sackman and Kelly[11] developed the expression for tuning parameter, and Igusa and Der Kiureghian[14] obtained the expressions for interaction, spatial coupling and non-classical damping parameters. These parameters have enabled a simple characterization of the complex behaviour of a combined system.

Several decoupled methods have also been developed in the past two decades for the estimation of internal forces in secondary systems in a simple and convenient manner. Depending on how accurately these methods have accounted for (i) interaction, i.e. feedback effects between the primary and secondary methods, (ii) non-classical damping of the combined system, and (iii) spatial coupling between the support excitations of the secondary system, particularly when there is tuning between the frequencies of the primary and secondary systems, this estimation of internal forces has been little to very accurate. Most of the earlier methods (e.g. those by Amin *et al*.,[17] Lin,[18] Vashi,[19] Thailer,[20] Wu *et al*.,[21] and Leimbach and Schmidt[22]) did not properly consider these special characteristics of the combined P–S systems and thus gave large errors in response evaluation. The use of 'interaction-free' spectra by Burdisso and Singh[23] and the formulation of cross-cross floor spectrum (CCFS) approach by Asfura and Der Kiureghian[24] and Saudy *et al*.[25] however offered significant improvements over these methods.

In both coupled and decoupled approaches, with a few exceptions (e.g. see References 26 and 27), the previous methods have been based on the direct use of design spectrum ordinates and, therefore, appropriate modal combination rules have been developed. These rules, besides being approximate, do not consider inherent variations in the design ground motion due to random phasing of the constituent waves. It is also not possible to obtain the higher-order response peak amplitudes from these methods, while the higher-order peaks may in fact play a significant role in damage to the secondary systems.[28] In contrast, a power spectral density function (PSDF)-based approach can be used to obtain the peak responses for desired orders of peaks with a given level of confidence. The key step of this approach is to obtain the PSDF of the desired response function by multiplying the squared modulus of the transfer function with the PSDF of the input ground motion (see References 29 and 30). By taking the moments of the response PSDF, and with the knowledge of the excitation duration, the root-mean-square (r.m.s) value, a_{rms}, of the response and peak factors may be determined. The largest and higher peak amplitudes of the response for a given confidence level are then obtained by multiplying a_{rms} with the corresponding peak factors.

This paper proposes the estimation of the stochastic seismic response of a linear multiply supported secondary system through a new formulation for the transfer function. The key feature of this formulation is that the transfer function of the desired response function has been obtained by describing the displacements of the classically damped primary and secondary systems in terms of their fixed-base modes. This concept was originally considered by Hurty[31] and Craig and Bampton[32] in the convenient displacement method analysis of complex structures, e.g. space vehicle structures, by using first few mode shapes of the sub-structures. Later, this was used by several researchers in the soil–structure interaction problems (see References 33–37), by Igusa *et al.*[38,39] in the analysis of continuous structural systems supporting a number of continuous subsystems with rigid, single-point connections, and recently by Gupta[40] in the generation of floor response spectra. Thus, the proposed formulation does not require the calculations of the combined system properties and also, all the special dynamic properties of the combined P–S system as mentioned earlier have been implicitly accounted for in the formulation. A few example P–S systems have been considered to illustrate the proposed formulation by obtaining the transfer functions of several response functions. The probabilistic estimates of some of these response functions have also been obtained in case of the USNRC[41] ground PSDF and compared with the results of time–history analyses.

2. FORMULATION OF THE TRANSFER FUNCTION

Consider a linear, classically damped, n-degree-of-freedom (DOF) primary system supporting a linear, classically damped, N-DOF secondary system as shown in Figure 1. Masses in both the sub-systems are assumed to be lumped at the respective degrees of freedom, i.e. $M_1, M_2, \ldots, M_n$ at the primary DOFs and $m_1, m_2, \ldots, m_N$ at the secondary DOFs. The first a DOFs of the primary system are assumed to be attached to the first a DOFs of the secondary system, through the elements, 1–1, 2–2, $\ldots$, a–a. These elements are, respectively, assumed to have stiffnesses, $K_1, K_2, \ldots, K_a$, and dampings, $C_1, C_2, \ldots, C_a$. The secondary system may also be supported on the ground. Let $\{X(t)\}$ denote the displacements along the DOFs of the primary system relative to the ground, and $\{u(t)\}$ denote the displacement vector for the secondary system. For the first a DOFs, the secondary system displacements are measured relative to the corresponding support DOFs of the primary system while the remaining DOFs are measured relative to the ground. The decoupled primary system is subjected to the excitation, $\ddot{z}(t)$, at its base, and to the interaction forces, $\{f(t)\}$, at its masses, M_1, $M_2, \ldots, M_a$ along the corresponding DOFs. The equations of motion for the primary system may be written as

$$[M]\{\ddot{X}(t)\} + [C]\{\dot{X}(t)\} + [K]\{X(t)\} = -[M]\{R\}\ddot{z}(t) + \{f(t)\} \tag{1}$$

where $[M]$, $[C]$, and $[K]$, respectively, represent the mass, damping and stiffness matrices of the primary system, and $\{R\}$ is the influence vector, denoting rigid-body displacements along the primary DOFs for the unit translation of the base. Since the two sub-systems are connected at a DOFs, the first a elements of $\{f(t)\}$ are the forces applied by the members connecting them and remaining $(n-a)$ elements are zeros. Let the kth element of $\{f(t)\}$ be given by $\vartheta_k(t)$ for $k = 1, 2, 3, \ldots, a$, where $\vartheta_k(t)$ is the interaction force acting along the kth primary DOF. Assuming the primary system to be classically damped and expanding its response in terms of the orthonormal mode shapes of the system, we may write

$$\{X(t)\} = [\phi]\{q(t)\} \tag{2}$$

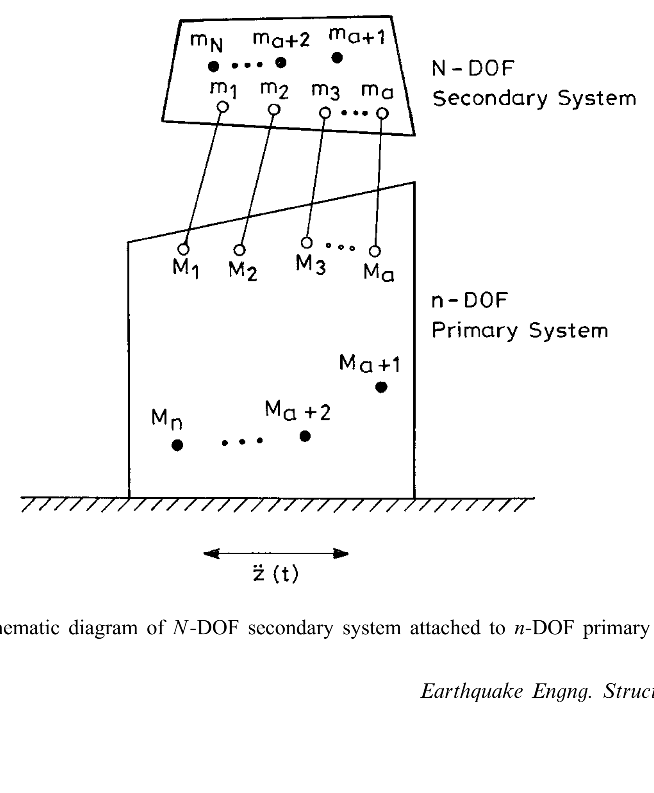

Figure 1. Schematic diagram of N-DOF secondary system attached to n-DOF primary system

where $[\phi]$ is the real-valued modal matrix of the primary system and $\{q(t)\}$ is the vector of the normal co-ordinates. On using the orthogonality relationships, the decoupled equation for the rth primary mode is given by

$$\ddot{q}_r(t) + 2\zeta_r\omega_r\dot{q}_r(t) + \omega_r^2 q_r(t) = -\alpha_r\ddot{z}(t) + \{\phi^{(r)}\}^{\mathrm{T}}\{f(t)\}, \quad r = 1, 2, \ldots, n \tag{3}$$

where $\{\phi^{(r)}\}^{\mathrm{T}}$ is the transpose of the rth mode shape, $\{\phi^{(r)}\}$, α_r is the modal participation factor given by $\{\phi^{(r)}\}^{\mathrm{T}}[M]\{R\}$, and ω_r and ζ_r, respectively, denote the natural frequency and damping ratio in the rth mode of the primary system. The last term in equation (3) represents the feedback from the secondary system to the primary system in the rth mode and this may be written as $\sum_{k=1}^{a}\phi_k^{(r)}\vartheta_k(t)$ where, $\phi_k^{(r)}$ denotes the kth element of $\{\phi^{(r)}\}$. On Fourier-transforming equation (3), we get

$$q_r(\omega) = H_r(\omega)\left(-\alpha_r\ddot{z}(\omega) + \sum_{k=1}^{a}\phi_k^{(r)}\vartheta_k(\omega)\right), \quad r = 1, 2, \ldots, n \tag{4}$$

where

$$H_r(\omega) = \frac{1}{\omega_r^2 - \omega^2 + 2\mathrm{i}\zeta_r\omega_r\omega}, \quad r = 1, 2, \ldots, n \tag{5}$$

is the modal transfer function relating the displacement of the single-degree-of-freedom (SDOF) oscillator in the rth mode to the input ground acceleration, and $q_r(\omega)$, $\ddot{z}(\omega)$ and $\vartheta_k(\omega)$, respectively, are the Fourier transforms of the corresponding time-dependent variables, $q_r(t)$, $\ddot{z}(t)$ and $\vartheta_k(t)$. It is possible to further write $\vartheta_k(\omega)$ as

$$\vartheta_k(\omega) = (\mathrm{i}\omega C_k + K_k)u_k(\omega), \quad k = 1, 2, \ldots, a \tag{6}$$

where C_k and K_k, respectively, are the damping and stiffness of the element connecting the secondary system to the primary system at the kth DOF, and $u_k(\omega)$ is the Fourier transform of the displacement, $u_k(t)$, for the kth secondary DOF.

On Fourier-transforming equation (2) and substituting $q_r(\omega)$ from equation (4), we can express the displacement for the jth primary DOF in frequency domain as

$$X_j(\omega) = \sum_{r=1}^{n}\phi_j^{(r)}H_r(\omega)\left(-\alpha_r\ddot{z}(\omega) + \sum_{k=1}^{a}\phi_k^{(r)}\vartheta_k(\omega)\right), \quad j = 1, 2, \ldots, n \tag{7}$$

Substituting in equation (7), the expression for $\vartheta_k(\omega)$ as in equation (6), we obtain

$$X_j(\omega) = \sum_{r=1}^{n}\phi_j^{(r)}H_r(\omega)\left(-\alpha_r\ddot{z}(\omega) + \sum_{k=1}^{a}\phi_k^{(r)}(\mathrm{i}\omega C_k + K_k)u_k(\omega)\right), \quad j = 1, 2, \ldots, n \tag{8}$$

This equation describes how the primary system displacements depend on the stiffness and damping properties and on the relative displacements in the members connecting the primary system with the secondary system. Similar relationships should exist for the explicit dependence of the secondary system displacements on the primary system displacements. For this, we proceed by following the same steps as used in equations (1)–(8).

The equations of motion for the decoupled secondary system (including the connecting members, 1–1, 2–2, $\ldots$, a–a) may be written as

$$[m]\{\ddot{u}(t)\} + [c]\{\dot{u}(t)\} + [k]\{u(t)\} = \{F(t)\} \tag{9}$$

where $[m]$, $[c]$ and $[k]$, respectively, are the mass, damping and stiffness matrices of the fixed-base secondary system (i.e. the secondary system obtained by completely restraining the far ends of the members connecting

the secondary system with the primary system and ground). $\{F(t)\}$ is the vector of input excitation to the secondary system due to the motions at the far ends of the connecting members, and is given by

$$\{F(t)\} = -[\tilde{m}]\{\ddot{\tilde{X}}(t)\} - [\tilde{c}]\{\dot{\tilde{X}}(t)\} - [\tilde{k}]\{\tilde{X}(t)\} - [m]\{r\}\ddot{z}(t) \tag{10}$$

Here $[\tilde{m}]$, $[\tilde{c}]$ and $[\tilde{k}]$ are the matrices of $N \times a$ dimension. The element, (i, j) of $[\tilde{m}]$ is equal to the element, (i, j) of $[m]$ for $i = j$ and is equal to zero otherwise. The matrices, $[\tilde{c}]$ and $[\tilde{k}]$, are obtained by considering the first a columns of the matrices, $[c]$ and $[k]$, respectively, and by considering the dampings and stiffnesses of the members connecting the primary and secondary systems, i.e. $C_1, C_2, \ldots, C_a$, and $K_1, K_2, \ldots, K_a$, to be zero. Further, the vectors $\{\ddot{\tilde{X}}(t)\}$, $\{\dot{\tilde{X}}(t)\}$ and $\{\tilde{X}(t)\}$, constitute the first a elements of $\{\ddot{X}(t)\}$, $\{\dot{X}(t)\}$ and $\{X(t)\}$ respectively, and $\{r\}$ is the influence vector relating the DOFs of the secondary system to the base motion. Again, on using the normal-mode approach, we can write

$$\{u(t)\} = [\psi]\{\eta(t)\} \tag{11}$$

where $[\psi]$ is the modal matrix of the fixed-base secondary system and $\{\eta(t)\}$ is the vector of the normal co-ordinates. Then, the decoupled equation for the lth secondary mode is given by

$$\ddot{\eta}_l(t) + 2\xi_l\Omega_l\dot{\eta}_l(t) + \Omega_l^2\eta_l(t) = \{\psi^{(l)}\}^{\mathrm{T}}\{F(t)\}, \quad l = 1, 2, \ldots, N \tag{12}$$

where $\{\psi^{(l)}\}^{\mathrm{T}}$ is the transpose of the lth modal vector, $\{\psi^{(l)}\}$, and Ω_l and ξ_l, respectively, denote the natural frequency and damping ratio of the lth mode of the secondary system. On Fourier-transforming equation (12), we obtain

$$\eta_l(\omega) = h_l(\omega)\sum_{i=1}^{N} \psi_i^{(l)} F_i(\omega), \quad l = 1, 2, \ldots, N \tag{13}$$

where $\eta_l(\omega)$ and $F_i(\omega)$, respectively, are the Fourier transforms of $\eta_l(t)$ and $F_i(t)$ (i.e. the ith element of $\{F(t)\}$), and $\psi_i^{(l)}$ represents the ith element of $\{\psi^{(l)}\}$. The modal transfer function relating the displacement of the lth SDOF system oscillator to the input ground acceleration is given by

$$h_l(\omega) = \frac{1}{\Omega_l^2 - \omega^2 + 2\mathrm{i}\xi_l\Omega_l\omega}, \quad l = 1, 2, \ldots, N \tag{14}$$

Now, on Fourier-transforming equation (10), it is possible to express the ith element of $\{F(t)\}$ in frequency domain as

$$F_i(\omega) = -m_{ii}r_i\ddot{z}(\omega) + \sum_{j=1}^{a} X_j(\omega)A_{ij}(\omega), \quad i = 1, 2, \ldots, N \tag{15}$$

with

$$A_{ij}(\omega) = \omega^2\tilde{m}_{ij} - \mathrm{i}\omega\tilde{c}_{ij} - \tilde{k}_{ij} \tag{16}$$

denoting the transfer function between $F_i(t)$ (i.e. the ith element of input excitation to the secondary system) and the displacement, $X_j(t)$, along the jth primary DOF. In equations (15) and (16), m_{ii} is the ith diagonal element of $[m]$, r_i is the ith element of the influence vector, $\{r\}$ and $\tilde{m}_{ij}$, $\tilde{c}_{ij}$ and $\tilde{k}_{ij}$, respectively, denote the element, (i, j) of the matrices $[\tilde{m}]$, $[\tilde{c}]$ and $[\tilde{k}]$. Substitution of equation (15) into the expression for $\eta_l(\omega)$ (see equation (13)) now leads to

$$\eta_l(\omega) = h_l(\omega)\left(-\sum_{i=1}^{N}\psi_i^{(l)}m_{ii}r_i\ddot{z}(\omega) + \sum_{i=1}^{N}\sum_{j=1}^{a}\psi_i^{(l)}A_{ij}(\omega)X_j(\omega)\right), \quad l = 1, 2, \ldots, N \tag{17}$$

Equation (11) may now be Fourier-transformed, and with the substitution of equation (17), the following expression may be obtained for the pth displacement of the secondary system in frequency domain:

$$u_p(\omega)=\sum_{l=1}^{N}\psi_p^{(l)}h_l(\omega)\left(-\sum_{i=1}^{N}\psi_i^{(l)}m_{ii}r_i\ddot{z}(\omega)+\sum_{i=1}^{N}\sum_{j=1}^{a}\psi_i^{(l)}A_{ij}(\omega)X_j(\omega)\right),\quad p=1,2,\ldots,N \tag{18}$$

This equation describes how the secondary system displacements depend on the motions along the primary DOFs and shows that this dependence is not much different for the *vice versa* dependence of the primary system displacements as described by equation (8). Now, on substituting $X_j(\omega)$ from equation (8) into equation (18) and simplifying, we obtain

$$\begin{aligned}u_p(\omega)=\sum_{l=1}^{N}\sum_{s=1}^{N}&\left[-\psi_p^{(l)}\psi_s^{(l)}h_l(\omega)m_{ss}r_s\ddot{z}(\omega)+\sum_{j=1}^{a}\psi_p^{(l)}\psi_s^{(l)}h_l(\omega)A_{sj}(\omega)\sum_{r=1}^{n}\phi_j^{(r)}H_r(\omega)\right.\\&\left.\times\left(-\alpha_r\ddot{z}(\omega)+\sum_{k=1}^{a}\phi_k^{(r)}(\mathrm{i}\omega C_k+K_k)u_k(\omega)\right)\right],\quad p=1,2,\ldots,N\end{aligned} \tag{19}$$

Further rearrangement of the terms in equation (19) leads to the following N linear, simultaneous equations for the N unknown secondary system displacements at a given frequency, ω,

$$\begin{aligned}&u_p(\omega)-\sum_{k=1}^{a}\left(\sum_{l=1}^{N}\sum_{s=1}^{N}\sum_{j=1}^{a}\sum_{r=1}^{n}\psi_p^{(l)}\psi_s^{(l)}\phi_j^{(r)}\phi_k^{(r)}h_l(\omega)H_r(\omega)A_{sj}(\omega)(\mathrm{i}\omega C_k+K_k)\right)u_k(\omega)\\&\quad=-\left[\sum_{l=1}^{N}\sum_{s=1}^{N}\psi_p^{(l)}\psi_s^{(l)}h_l(\omega)\left\{m_{ss}r_s+\sum_{j=1}^{a}\sum_{r=1}^{n}A_{sj}(\omega)\phi_j^{(r)}H_r(\omega)\alpha_r\right\}\right]\ddot{z}(\omega),\quad p=1,2,\ldots,N\end{aligned} \tag{20}$$

Due to the linear dependence of these N unknowns on the input ground motion, $\ddot{z}(\omega)$, the solution of these simultaneous equations will give the N transfer functions, each relating the displacement along a DOF of the secondary system to the input ground acceleration. These transfer functions can then be used in equation (8) to obtain the transfer functions for the primary system displacements, $X_1(t),X_2(t),\ldots,X_n(t)$. Due to the linear relationship of the other response quantities with the primary and secondary system displacements, their transfer functions can also be easily determined. To illustrate, let $\mathscr{H}_i^{\mathrm{P}}(\omega)$ and $\mathscr{H}_i^{\mathrm{S}}(\omega)$, respectively, denote the displacement transfer functions of the ith primary DOF and the ith secondary DOF. If the ith primary DOF happens to be a support DOF, then the transfer function for the displacement along the ith secondary DOF as measured relative to ground will be given by $\mathscr{H}_i^{\mathrm{P}}(\omega)+\mathscr{H}_i^{\mathrm{S}}(\omega)$. Further, if κ_i represents the stiffness of the member connecting the ith primary DOF to the ith secondary DOF, the transfer function for the force response in this member will simply be $\kappa_i\mathscr{H}_i^{\mathrm{S}}(\omega)$. Similarly, the transfer function for the force response in a member connecting the jth and kth secondary DOFs, will be given by $\kappa_{j-k}(\mathscr{H}_j^{\mathrm{S}}(\omega)-\mathscr{H}_k^{\mathrm{S}}(\omega))$ for $j,k=(a+1),(a+2),\ldots,N$ and by $\kappa_{j-k}(\mathscr{H}_j^{\mathrm{P}}(\omega)+\mathscr{H}_j^{\mathrm{S}}(\omega)-\mathscr{H}_k^{\mathrm{S}}(\omega))$ for $j=1,2,3,\ldots,a$ and $k=(a+1),(a+2),\ldots,N$, where κ_{j-k} denotes the stiffness of the member.

3. NUMERICAL EXAMPLES

3.1. Transfer functions

The proposed formulation for the transfer functions of the secondary system responses has been illustrated by considering two example P–S systems, A and B as shown in Figures 2 and 3, respectively. These idealized

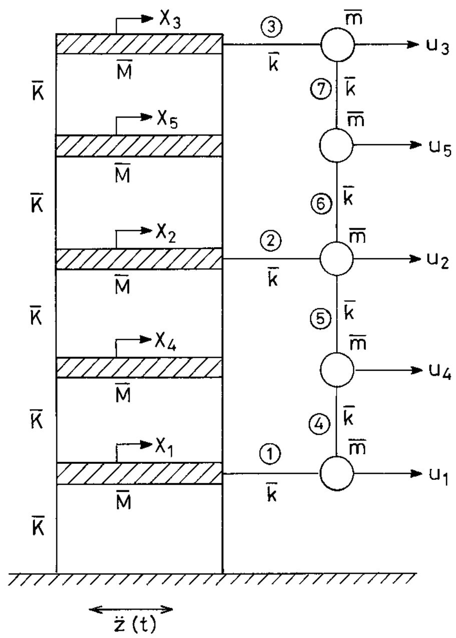

Figure 2. Idealized model of example System A

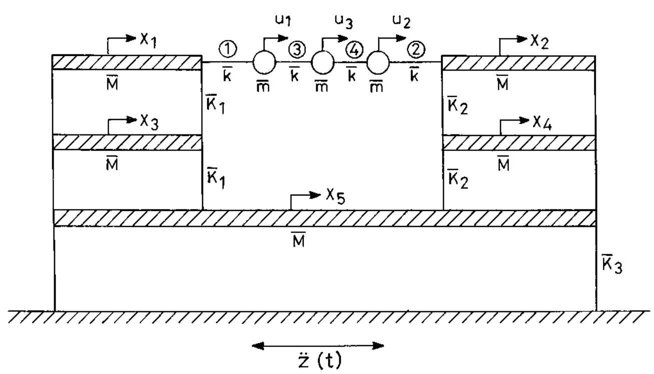

Figure 3. Idealized model of example System B

systems represent typical P–S systems encountered in practice and have also been examined earlier by Asfura and Der Kiureghian.[24] Different configurations adopted in these systems are expected to cover a wide variety of practical situations. For example, System B is representative of those cases where a secondary system is supported between two adjacent primary sub-systems.

In both of the systems considered, the vertical and rotational degrees of freedom have been assumed to be of little significance. Thus, System A consists of a 5-DOF primary system and a 5-DOF secondary system with three attachment points. For the primary system, uniform floor masses and interstory stiffnesses have been considered with $\bar{M} = 1{\cdot}0 \times 10^5$ kg and $\bar{K} = 2{\cdot}0 \times 10^4$ kN/m. System B consists of a 5-DOF primary system and a 3-DOF secondary system with two attachment points. The primary system has been assumed to have equal floor masses of value, $\bar{M} = 1{\cdot}0 \times 10^5$ kg. The interstorey stiffnesses have been taken as $\bar{K}_1 = 2{\cdot}0 \times 10^4$ kN/m, $\bar{K}_2 = 3{\cdot}0 \times 10^4$ kN/m and $\bar{K}_3 = 5{\cdot}0 \times 10^5$ kN/m. The mass and stiffness properties of the secondary systems

Table I. Natural frequencies of example systems (in rad/sec)

Mode no.	System A		System B	
	Primary	Secondary	Primary	Secondary
1	4·03	4·02	8·60	8·59
2	11·75	5·59	10·49	15·87
3	18·52	8·49	22·75	20·74
4	23·79	9·68	27·79	—
5	27·14	11·41	74·34	—

in each of the two example systems have been considered to be uniform. In System A, the ratio $\bar{k}/\bar{m}$ has been taken as $31{\cdot}22\,\mathrm{sec}^{-2}$. This corresponds to a tuned P–S system as the fundamental frequencies of the decoupled primary and secondary systems are same and also, the second frequency of the primary system is very close to the fifth frequency of the secondary system. In System B also, the ratio, $\bar{k}/\bar{m}$, kept at $126{\cdot}00\,\mathrm{sec}^{-2}$ causes matching of the fundamental frequencies of the decoupled primary and secondary systems. The natural frequencies of both primary and secondary systems in Systems A and B have, respectively, been given in Table I. In these systems, the damping ratio has been assumed to be 0·05 for all the modes in the primary system and 0·02 for all the modes in the secondary system.

To illustrate the effect of mass ratio on the transfer functions of various response functions, in each example system, three different values of mass ratio, $\bar{m}/\bar{M} = 0{\cdot}0002, 0{\cdot}02$ and $0{\cdot}2$ have been considered corresponding to the cases of light, moderately heavy and very heavy secondary systems. Transfer functions have been obtained for different displacement and force responses in the two example systems by using the proposed formulation, and the modulii of these functions have been shown in Figures 4–9. For System A, displacement transfer functions have been obtained for the $(X_2 + u_2)$ and u_5 responses (i.e. the displacements for the second and fifth DOFs as measured relative to the ground) as shown in Figures 4 and 5, and the force transfer functions for the attached member 1 (to the primary system) and unattached member 7 have been obtained as shown in Figures 7 and 8. For System B, the displacement transfer functions for the $(X_2 + u_2)$ response (i.e. the displacement for the second DOF as measured relative to the ground) and the transfer functions for the force response in the unattached member 3 have been shown in Figures 6 and 9, respectively. Exactly same curves as in these figures are obtained via the formulation based on the exact calculation of the combined system eigenproperties. Details of such a formulation, while using the state-space approach,[1] may be found elsewhere (e.g. see References 42 and 40). It may be noted that the proposed formulation is an exact formulation as no approximations have been made regarding the interaction forces between the primary and secondary systems. The fact that the displacements of both systems have been described in terms of their mode shapes is not a limitation because any p-dimensional vector can be expressed in terms of p linearly independent vectors which span a p-dimensional vector space. We may need complex mode shapes for describing the $(n + N)$-dimensional vector of combined system displacements, but the n and N-dimensional vectors for the displacements of the individual sub-systems may still be expressed as linear combinations of their real mode shapes due to their linear independence.

It is observed in Figures 4–6 that for any displacement response, the transfer function modulus curves for the different mass ratios are in excellent agreement except for the frequency range extending upto the highest frequency of the secondary system. Thus, the relative lightness or heaviness of the secondary system makes difference only for those excitation frequencies which are within the range of the natural frequencies of the secondary system. This should however be true only when the highest secondary system frequency is lower than the highest primary system frequency as in the two examples cases considered here. For very stiff secondary systems, this may not be the case and then, the mass ratio may influence the transfer function only for those excitation frequencies which are within the natural frequencies of the primary system. In view

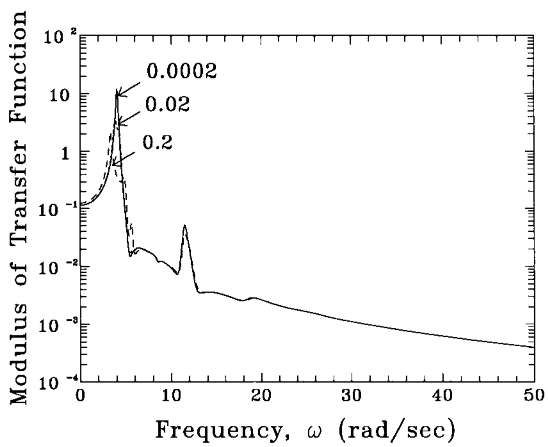

Figure 4. Modulii of transfer functions for $(u_2 + X_2)$ response of System A in case of different mass ratios

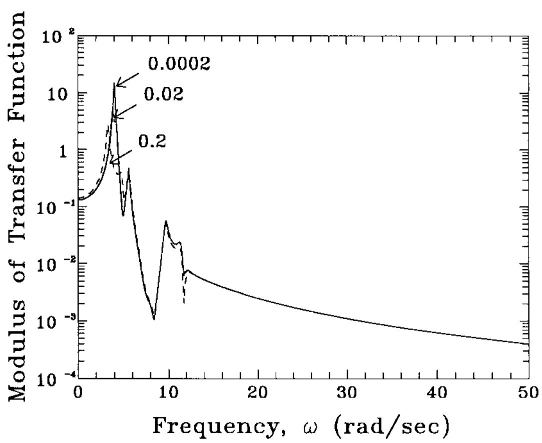

Figure 5. Modulii of transfer functions for u_5 response of System A in case of different mass ratios

of this, if the earthquake excitation has substantial energy outside the ‘overlap’ range of the primary and secondary system frequencies, one may possibly ignore the feedback effects between the two sub-systems and carry out the response calculations by a simple decoupled analysis. Further study of the curves in Figures 4–6 shows that for the mass ratio = 0·0002, the curves exhibit a single sharp peak at the fundamental frequency of the two sub-systems, while for the mass ratio = 0·2, two distinct peaks of much lower magnitude are observed in the vicinity of this fundamental frequency. This is due to the varying degrees of interaction in the three cases. When the secondary system is light, there is little interaction despite tuning between the fundamental frequencies of the two sub-systems. However, this should correspond to a much larger displacement response as the peak in this case is as much as five times higher than the two peaks for heavy secondary systems. This shows the degree of conservatism involved in ignoring the ‘feed-back effects’ in case of heavy secondary systems. It is also to be noted that peaks do not show up for all the frequencies of the primary and secondary systems. On examining Figure 4, we find that in the curves for mass ratios equal to 0·0002 and 0·02, there is no peak corresponding to the second secondary mode while a peak is displayed in the curve for mass ratio = 0·2. This observation is however not true for Figure 5 where the second secondary mode appears to

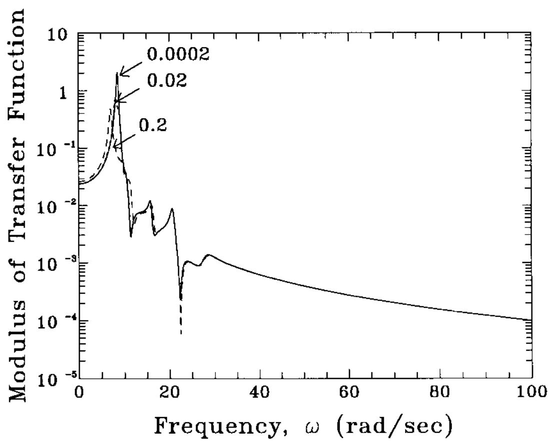

Figure 6. Modulii of transfer functions for $(u_2 + X_2)$ response of System B in case of different mass ratios

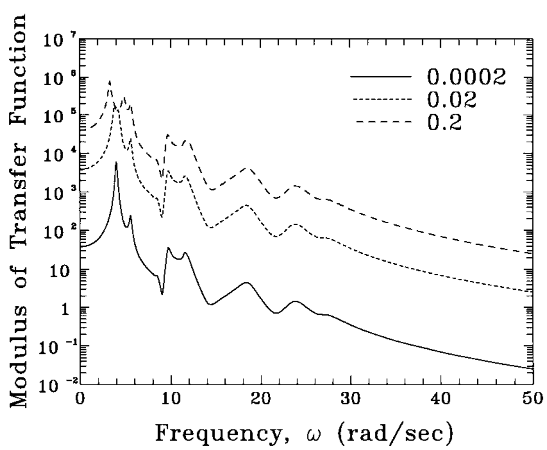

Figure 7. Modulii of transfer functions for force in member 1 of System A in case of different mass ratios

play a significant role for all mass ratios. Thus, depending upon the response quantity of a system and the degree of interaction between the two sub-systems, a given mode of primary or secondary system may or may not be excited.

Similar observations can also be made for the force transfer function curves as shown in Figures 7–9. Here, many more and flatter peaks are seen to appear as compared to the displacement transfer function curves. This is as expected since the participation of the higher modes is generally greater for the force response than for the displacement response. Further, Figure 8 clearly shows how the transition takes place from a single sharp peak in case of light secondary systems to the two distinct peaks in case of heavy secondary systems. In contrast with the displacement transfer function curves, however, here the curves for the mass ratio equal to 0·2 show the maximum steady-state responses at all excitation frequencies while those for the mass ratio equal to 0·0002 consistently show the minimum responses. This is so since the natural frequencies of the primary and secondary systems for all the three different mass ratios are kept unchanged, and therefore, higher mass ratio is associated with higher stiffness and thus higher response of the constituent members. Within the 'overlap' frequency range, the lower mass ratios lead to greater displacement responses as discussed above, and thus

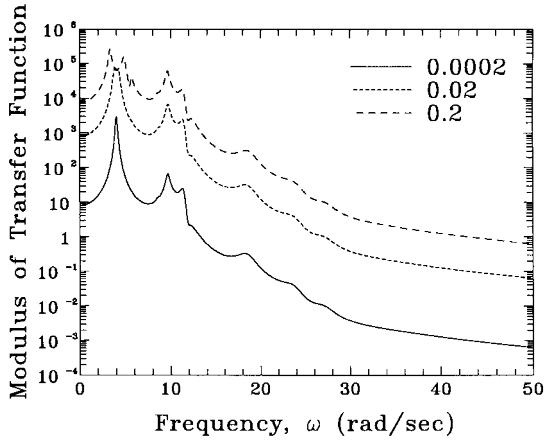

Figure 8. Modulii of transfer functions for force in member 7 of System A in case of different mass ratios

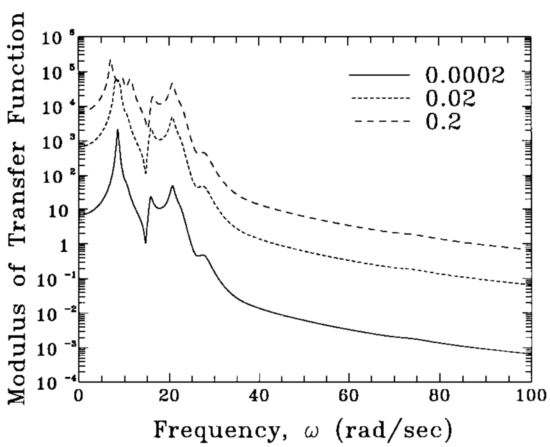

Figure 9. Modulii of transfer functions for force in member 3 of System B in case of different mass ratios

the force responses for different mass ratios are found to be closer in this range. Outside this range, i.e. for the higher frequencies, however, the displacement responses are same for the three mass ratios, and thus, the three curves are found to be 'parallel' to each other. This is consistent with the same frequency-independent ratios of 10 and 1000 which will be maintained between the force responses for the mass ratio of 0·2 and the mass ratios of 0·02 and 0·0002 at these frequencies.

It may be noted that the example systems A and B do not consider those cases where a secondary system is multiply supported on the primary system as well as on the ground. It has however been seen by considering an example case of this type also that the proposed formulation gives exact transfer functions and that the above trends are generally true.

3.2. Stochastic estimates

A PSDF-based approach is implicitly based on the assumption that the input excitation process is stationary and that no additional non-stationarity is introduced into the response process due to the sudden application of the excitation. Hence, it has been an accepted practice to consider the PSDF of a fictitious 'equivalent

stationary' ground motion process such that it is compatible with the specified design spectrum within the assumption of stationarity for the response process. Such a PSDF used to characterize the seismic ground motion is called as the spectrum-compatible PSDF and is determined from the given design spectrum by using iterative schemes, e.g. those by Kaul,[43] Sundararajan,[44] Unruh and Kana[45] and Pfaffinger.[46] However, the design spectra obtained for different damping ratios do not correspond to the same spectrum-compatible PSDF. This is partly due to the lack of inherent compatibility between the design spectra corresponding to different damping ratios after the averaging and smoothing operations, and partly due to ignoring the nonstationarity in response due to finite operating time of the excitation process. The iterative scheme proposed by Shrikhande and Gupta[47] for spectrum-compatible PSDFs accounts for the effects of response nonstationarity as it is based on the use of time-dependent transfer function for the response process. Further, by using the 'envelope PSDF', i.e. the PSDF which envelops the PSDFs corresponding to the design spectra for different damping ratios, one can conveniently obtain the conservative stochastic responses of a P-S system with non-uniform modal damping ratios. It is also possible to directly use a design PSDF, e.g. that specified by USNRC,[41] to characterize the design ground motion.

The use of the proposed formulation within the framework of a PSDF-based stochastic approach has been illustrated by considering (i) System A with $\bar{m}/\bar{M} = 0{\cdot}02$, 5 per cent damping in primary system modes and 6 per cent damping in secondary system modes, and (ii) System B with $\bar{m}/\bar{M} = 0{\cdot}2$, 5 per cent damping in the modes of both primary and secondary systems, and with the design PSDF as given by USNRC[41] for a peak ground acceleration of $1{\cdot}0g$. It is assumed that the effects of non-stationarity are not required to be explicitly accounted for with this PSDF and that the design ground motion has a stationary duration of 24·42 sec. For relating the r.m.s. and the peak values of the response functions, the peak factors based on the order statistics formulation as in References 48–50 have been used. The responses for the 5 and 95 per cent probabilities of exceedance and 'expected' responses have been obtained for the displacements of all the secondary DOFs, as measured relative to ground, and for the forces in all the secondary system members. It may be noted that the modulii of the transfer functions of some of these responses have been shown earlier in Figures 4–9. As shown in Tables II and III, respectively, for the Systems A and B, the probabilistic estimates have been compared with those of the time-history analyses obtained by using the algorithm of Singh and Ghafory-Ashtiany.[51]

The accelerogram for the time-history analyses has been obtained by first obtaining the 5 per cent damping, mean response spectrum corresponding to the given ground PSDF and then by using the method of Shrikhande and Gupta[47] to generate the spectrum-compatible accelerogram. Since this accelerogram cannot be compatible with the response spectrum curves for the other damping ratios, deviations in the modal damping ratios of the combined system (beyond 0·05) have been accounted for by scaling the input accelerogram for each mode of the combined system. The scaling factor used for this purpose in the rth mode is the ratio of the response spectrum amplitude obtained from the design PSDF corresponding to the damping ratio, $\tilde{\zeta}_r$ (i.e. the damping ratio of the combined system in the rth mode) and at the frequency, $\tilde{\omega}_r$ (i.e. the natural frequency of the combined system in the rth mode) to the peak response of a SDOF oscillator of damping ratio, $\tilde{\zeta}_r$ and frequency, $\tilde{\omega}_r$ subjected to the spectrum-compatible accelerogram. It may be noted that the use of this scaling factor guarantees the perfect matching of the combined system response in the rth mode with that of the rth SDOF oscillator only when the system is classically damped. In a non-classically damped system, this is true only in an approximate sense because the modal damping ratios and natural frequencies of such a system are basically 'equivalent' parameters of a non-synchronous motion and thus do not exactly characterize the modal response time histories as they would do in case of a SDOF oscillator with the same properties. However, the example systems are mildly non-classical and therefore, this approach may be acceptable.

It may be observed from Tables II and III that the results of the time-history analyses are within the 5 and 95 per cent confidence level estimates and are generally in very good agreement with the mean responses from the stochastic analyses. For the reasons discussed above, this agreement is likely to become worse for the moderately or highly non-classical systems.

Table II. Comparison of the probabilistic estimates with time-history results for System A

Response function	Probabilistic estimates 95%	Expected	5%	Time history
D-1 (m)	1·817	2·228	2·851	1·873
D-2 (m)	2·237	2·750	3·527	2·408
D-3 (m)	2·205	2·517	3·230	2·261
D-4 (m)	2·660	3·268	4·191	2·760
D-5 (m)	2·802	3·443	4·414	3·034
F-1 (kN)	108·6	133·5	171·2	114·5
F-2 (kN)	128·0	157·3	201·7	140·6
F-3 (kN)	109·0	133·9	171·8	121·3
F-4 (kN)	53·9	66·2	84·9	55·7
F-5 (kN)	33·4	40·8	52·0	37·2
F-6 (kN)	36·9	45·3	58·0	41·9
F-7 (kN)	51·6	63·2	80·9	58·2

Note: Here, D-1, D-2, D-3, D-4 and D-5, respectively, represent the peak displacements (measured relative to ground) corresponding to u_1, u_2, u_3, u_4 and u_5; F-1, F-2, F-3, F-4, F-5, F-6 and F-7, respectively, represent the forces in the members 1, 2, 3, 4, 5, 6 and 7 of the secondary system

Table III. Comparison of the probabilistic estimates with time-history results for System B

Response function	Probabilistic estimates 95%	Expected	5%	Time history
D-1 (m)	0·565	0·702	0·874	0·724
D-2 (m)	0·473	0·590	0·735	0·604
D-3 (m)	0·662	0·821	1·022	0·820
F-1 (kN)	887·2	1095·7	1358·7	1135·2
F-2 (kN)	985·2	1219·3	1514·4	1253·6
F-3 (kN)	256·3	316·9	393·3	322·3
F-4 (kN)	538·5	662·8	819·8	672·9

Note: Here, D-1, D-2 and D-3, respectively, represent the peak displacements (measured relative to ground) corresponding to u_1, u_2 and u_3; F-1, F-2, F-3 and F-4, respectively, represent the forces in the members 1, 2, 3 and 4 of the secondary system

4. CONCLUSIONS

A PSDF-based stochastic approach has been presented for the seismic analysis of linear, multiply supported secondary systems. Central to this approach is an exact formulation of the transfer function of the desired response quantity which has been done by using the fixed-base modes of the primary and secondary systems. In this formulation, transfer functions for all the secondary system displacements are determined by repeated solution of N simultaneous equations for different frequencies of excitation. Since the eigenproperties of the combined system are not computed, the problems and inconvenience associated with the coupled approaches are completely avoided in the proposed formulation. By considering two example systems, typical of the

P–S systems frequently encountered in practice, the proposed formulation has been illustrated for various response functions. Also, the illustration of the stochastic approach based on this formulation has shown that the proposed approach can indeed be used to obtain reasonable estimates of secondary systems without treating the combined system as a single unit and thus, by retaining the practical advantages of a decoupled analysis. Moreover, it has all the advantages of a PSDF-based approach as elaborated in the introduction to this paper.

The proposed formulation is strictly applicable to those cases where there is negligible interaction of the primary system foundation with the surrounding soil. This will be extended to include the effects of soil-structure interaction in a forthcoming paper by the authors.

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